3 Embedding numeracy activities

Surveys of the numeracy needs of employers suggest that a fundamental requirement is the ability to handle quantitative problems which arise naturally in the course of work activities. These problems may be non-routine, and require the selection of suitable solution strategies. New technology is likely to be used in the collection and processing of data in order to solve the problem. The results will need to be analysed and presented in a format which is suitable for decision making.

It is not surprising that students undertaking vocational courses are most motivated when they are presented with realistic problem solving tasks of the type which might arise at work, including appropriate data collection. Students appreciate the relevance of the tasks, and see these as interesting and worthwhile training activities.

A strong case can be made for integrating numeracy activities into vocational courses where possible, as this can provide an effective and motivating way of meeting the training needs of both employers and students. A broad range of numeracy skills can be developed, including not only mathematical techniques but also problem solving, communication, use of information technology systems, and working with others. Autonomy in choosing a method of solution, and inclusion of practical data collection, are both known to increase student motivation and engagement with the learning activity. A further benefit is that detailed numerical investigation and analysis can give a deeper insight into the theoretical concepts of a vocational course.

Many vocational students in Wales have been required to study Essential Skills modules in numeracy, communication and information technology. Rather than deliver these skills separately, we have found that a combined project can represent a more interesting and realistic challenge with a range of opportunities to produce evidence to meet the assessment criteria. This approach is likely to be most effective if the same tutor has responsibility for delivering the set of essential skills, and has knowledge of the students' vocational area, so that realistic workplace problem solving tasks can be devised.

A successful approach is to plan a combined essential skills project around a quantitative problem in a real world context. Students collect data, undertake calculations and analysis by means of computer software, produce a written report and give a presentation of their results and conclusions, working as a small team where appropriate. In this way, challenges are met in numeracy, communication, information technology, and working with others.

An approach which works well for many types of project is to use the sequence of stages familiar to systems analysts:

Analysis, finding out the exact requirements for the project, and the questions which need to be answered.

Design, deciding on suitable techniques of data collection, processing and analysis.

Implementation, carrying out the practical activities of collecting and processing the necessary data, then calculating results.

Evaluation. Interpreting the results and presenting them in formats appropriate for decision making.

In a large and complex project, this set of activities may be repeated as a cyclic process. Initial results form the basis for the next stage of planning and implementation.

In the sections which follow, we outline two integrated essential skills projects: **re-roofing a house**, and **planning a holiday in Europe**. Inevitably, different projects will require different mathematical skills, will involve the collection and analysis of different types of data, and would have different audiences for the presentation of final results. By combining more than one type of project, it can be possible to cover a range of skills, for example:

- handling both geometrical data (such as lengths, areas and volumes) and arithmetical data (such as money and time),
- collecting data by both practical measurement, and research from book and internet sources,
- presenting results both formally (such as to a client or manager) and informally (such as to work colleagues or a group of friends).

Re-roofing a building

Construction students are presented with the task of designing a roof with a dormer window for a small detached bungalow with a given floor plan, then completing this in Welsh slate. The specification is to draw up a schedule of materials required, plan the construction, estimate the cost of materials and labour, and produce a time schedule for the work. The construction plan should be delivered as a written report and verbal presentation to the client. Appropriate use is to be made of computer applications for calculation and presentation of results.

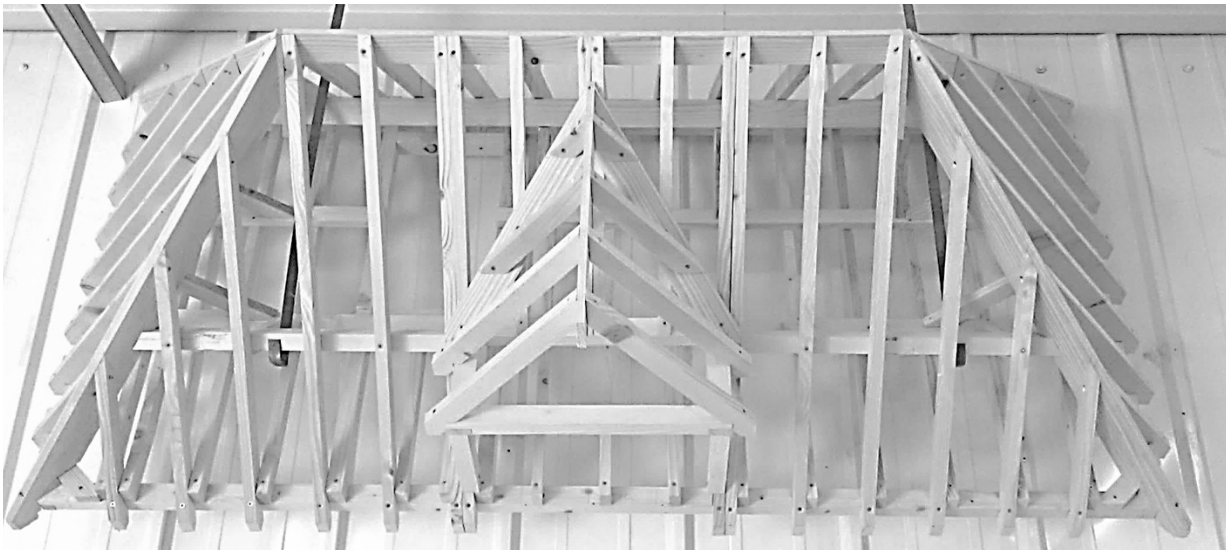


Figure 8: Model illustrating the structure for framing a timber roof

It is necessary to plan the geometry of the roof. Suitable sizes of timber need to be selected. Thought needs to be given to jointing the timber or using metal fixings. A suitable roof edge structure must be planned. The roof is to be cut on site, rather than constructed from factory-produced trusses.

Students have the opportunity to research information about roof structures from books or on-line sources

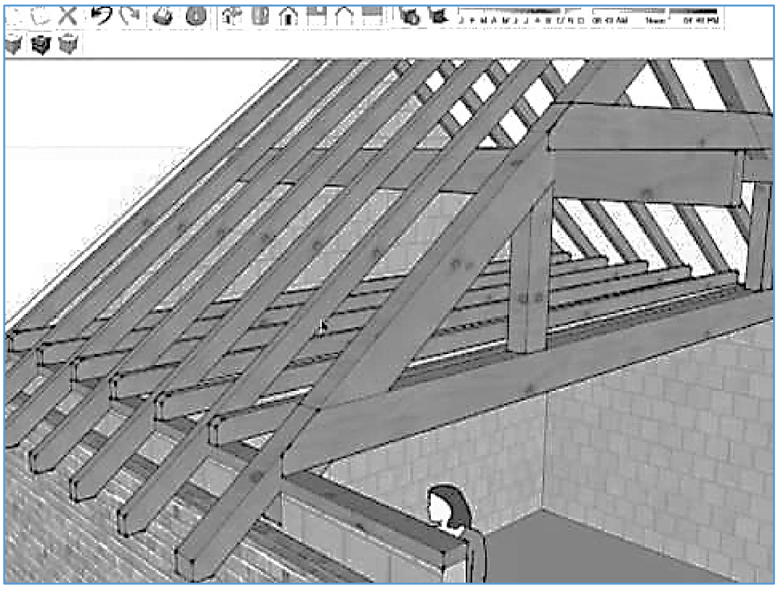


Figure 9: Video illustrating timber roof construction.
www.youtube.com/watch?v=n0CrtpuWL4w

Roof details will need to be planned, such as barge boards for gables, and facia boards for eaves. Guttering will need to be attached. A plan must be made for the construction of the roof frames. Materials can be tabulated, quantities estimated and costs calculated using a spreadsheet.

Design software can be used for laying out the geometry of the roof frames in plan and elevation views. Measurements can then be obtained for cutting the roof timbers. Lengths of timbers will depend on the chosen pitch angle for the roof.

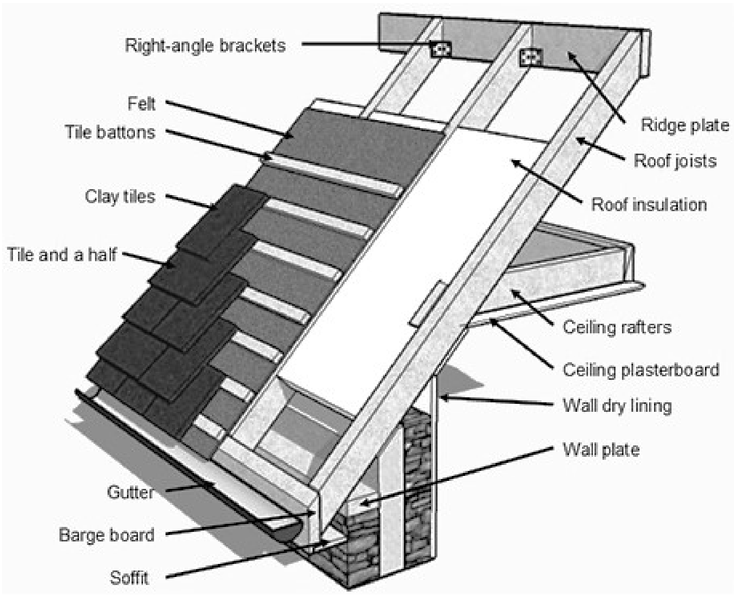


Figure 10: Roof construction
www.myhouseextension.com/roof.htm

Research can be carried out into the sizes and costs of roofing slates from local sources. Batons will need to be installed at appropriate intervals up the roof, above roof insulation and damp proofing material.

The roof can be split into a series of flat surfaces. The quantity of slates required can then be calculated for each surface, using area in square metres, the selected slate size and the overlap between slates.

Roofing costs can be calculated, including: slates, roofing battens, lead valley and flashings, waterproof membrane, and metal fittings. A final cost estimate will include labour and scaffolding. A commercial contractor will also need to allow for business overheads such as: fuel, premises, vehicle and machinery running costs, and down-time due to rain.

Holiday planning

For our second example of an integrated Essential Skills project, we will look at the planning of a holiday in Europe. Holiday planning can be a complex task involving decisions about:

- accommodation
- travel to the area
- places to visit
- travel within the area

Most of the necessary information is available on the Internet, but selecting from this mass of data can require a range of numeracy skills.

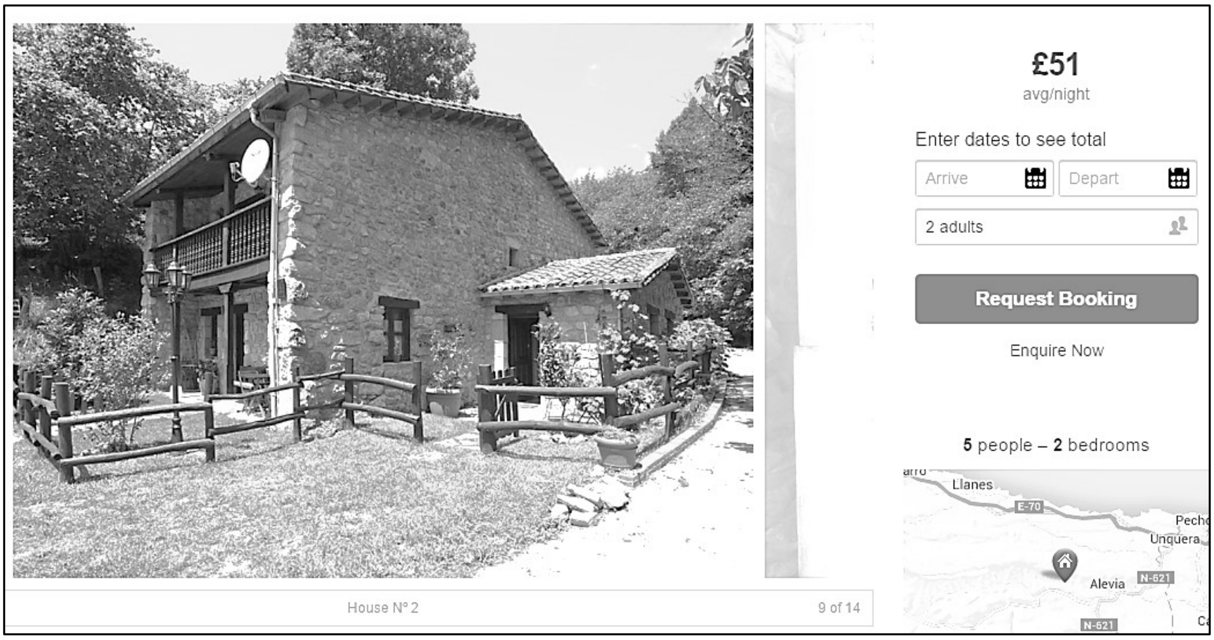
Students respond best if given autonomy to make their own planning decisions, so it is best not to over-specify the project requirements. For this task, students are simply asked to plan a holiday to the Picos De Europa region of North Spain for a week at Easter for a group of three friends, choosing whatever itinerary they wish. They are asked to estimate the costs of travel - to and within the area, accommodation, food and any other expenses.

It is unlikely that the students will already be familiar with this holiday area, so research using the internet will probably be necessary before planning begins:

> Extending into Asturias and Cantabria, the Picos de Europa are amongst Europe's most rugged and dramatic mountain ranges and being right by the stunning coast these mountains offer a wealth of exciting holiday choices.
> Outdoor activities range from gentle rambling, pony trekking and mountain biking to hearty hiking or serious mountaineering. You can canoe down the Deva or Sella rivers or head to the coast for a spot of surfing, sea kayaking or serious sand castle construction...
> Visit the pilgrimage centre of Covadonga or walk the Cares Gorge.
> The little market town of Potes, well-supplied with shops, bars and restaurants, is a centre for mountain excursions or activities, and a bit further on the cable car at Fuente Dé takes you up into the peaks.
> *www.casas.co.uk/holiday-picos-de-europa*

Students are free to make any further decisions about travel and accommodation.

Availability of different types of accommodation, from camping to self-catering and hotels can be considered and discussed amongst the group. Students might compare the advantages and disadvantages of different options, including a choice between remaining in one location for the week or staying overnight in different locations.



www.ownersdirect.co.uk/accommodation/p8140131

Figure 11: Self-catering holiday accommodation

Planning could include an itinerary for the week's stay, with walks in the area and visits to places of interest.

Travelling from North Wales, the flight options at different regional airports can be explored. Choices can be made between using public transport, car rental, or a combination of these. Estimates of living costs should be made, using the currency exchange rate.

As a culmination of the project, the group should present their proposals for the holiday itinerary and costings in a meeting. Different options can be considered and agreement reached.

The students should then produce an action plan for making all necessary bookings, determine a suitable amount of currency to convert to Euros, and compile a schedule of journey times. Maps which will be required during the holiday can be downloaded from the internet. These materials can then be presented in the form of an e-portfolio with accompanying explanatory notes.

In the previous two example projects, we assumed that numeracy activities were being carried out as part of course in Essential Skills. It is frequently the case, however, that there are opportunities to develop student numeracy within the context of a main vocational course. We now look at effective ways in which numeracy tasks can be integrated with vocational learning.

A useful framework for introducing real world problems into mathematics teaching has been proposed by Tang, Sui, & Wang (2003) from work in China. Practitioner research during the current project has focussed on ways in which this framework could be successfully adapted for introduction into vocational courses at further education level. Five approaches are identified by Tang et al. for embedding numeracy applications and modelling: Extension; Special Subject; Investigation Report; Paper Discussion; and Mini Scientific Research. These approaches represent a progression from applications set by the teacher, through increasing student involvement in the solution of real world problems, to totally independent project work designed by students themselves.

Extension

In this approach, students who have been studying a mathematical topic are presented with an ill-defined real world problem where they need to seek out additional data for its solution. The features of an extension problem are:

- The task may be restricted in extent, not requiring a full mathematical analysis of a complete system.
- The problem is intended to arouse students' interest, to convince students that the mathematics involved in its solution is useful.
- To encourage students to be flexible in finding ways to solve application problems using mathematical techniques which are known to them.
- The teacher creates the structure for the problem, and the student finishes the process of changing the problem into mathematical form and obtaining a solution.
- The application problems should be clearly and accurately formulated. However, this approach cannot realistically reflect the creative and open nature of general mathematical modelling.

As an example, we consider the following question which might be given at the end of a study of trigonometry:

> A photographer is intending to travel on the London Eye, a large Ferris wheel in London. She wishes to take panoramic views across the city, but needs to be at least higher than the roof of the nearby County Hall building to do this. She would like to know how many minutes will be available for the photographic session.

In this case, the student should obtain actual data, or at least reasonable estimates, for the speed of rotation of the wheel, its diameter, and the height of the adjacent building. This might be found by use of the Internet. The student is then free to devise their own method for numerical, graphical or analytic solution of the problem.

London Eye

The **London Eye** is a giant Ferris wheel on the South Bank of the River Thames in London.

The structure is 443 feet (135 m) tall and the wheel has a diameter of 394 feet (120 m). When erected in 1999 it was the world's tallest Ferris wheel.

Passenger capsules

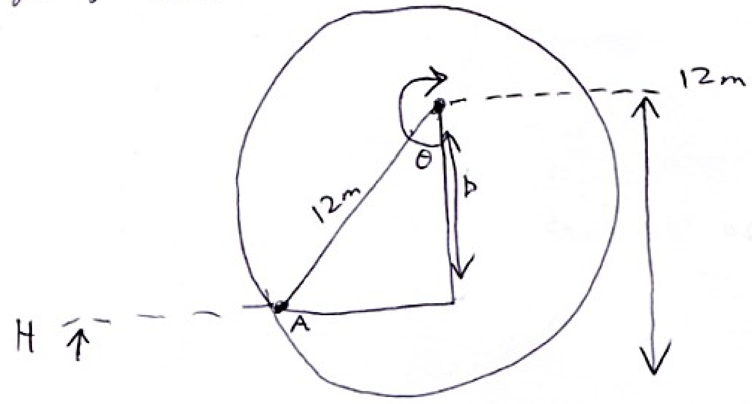
The wheel's 32 sealed and air-conditioned ovoidal passenger capsules, attached to the external circumference of the wheel and rotated by electric motors. The wheel rotates at 26 cm (10 in) per second (about 0.9 kph or 0.6 mph) so that one revolution takes about 30 minutes.

en.wikipedia.org/wiki/London_Eye

Figure 12: Information on the London Eye available from the Wikipedia website

Students are free to develop a solution which may involve sine or cosine methods. Either a numerical or graphical approach is possible, or results may be presented by both methods. This can develop a flexible approach by students to interchanging between arithmetical, algebraic and geometrical representations of data sets.

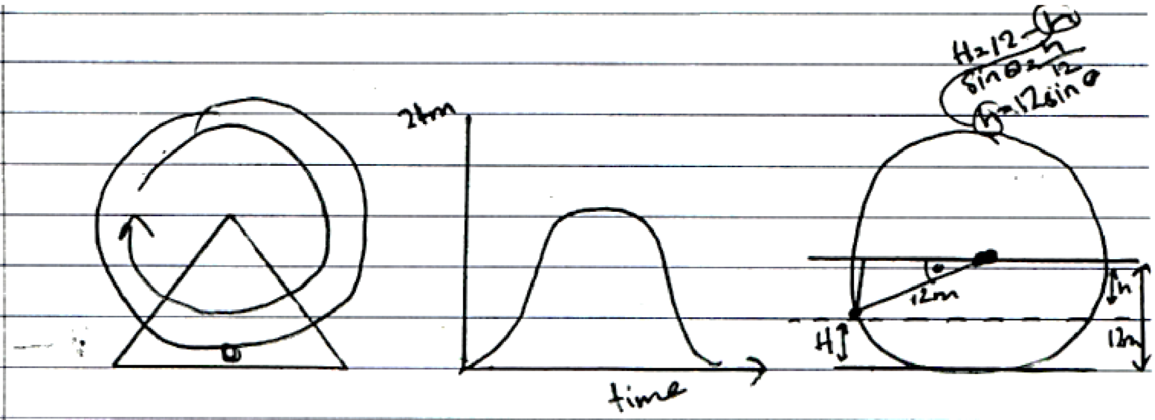
The Height of a car above the ground (H), will be given by: $H = 12\,m - b$



where: $\cos\theta = \frac{b}{12} \rightarrow b = 12 \times \cos\theta$

So for any angle θ, the height of the car (A) will be given by:

$H = 12 - 12 \times \cos\theta$



1		
2	0	= 12 - 12 * sin(A2*PI()/180)
3	= A2+10	
4		
5		

Figure 13: Student solutions to the London Eye numeracy task

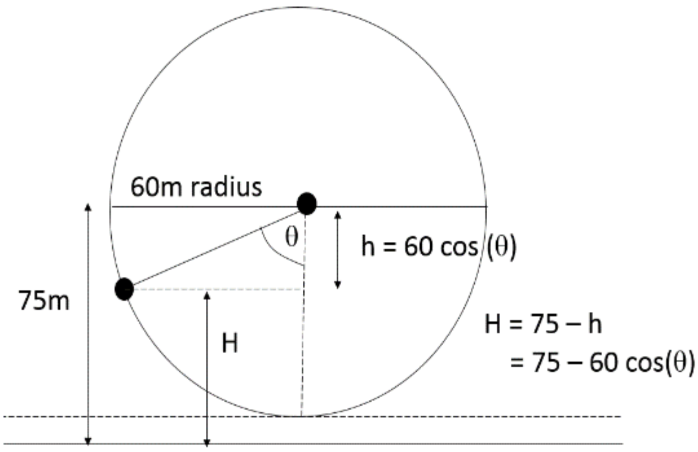


Figure 14: Developing an algebraic solution to the Ferris wheel problem

The wheel has a radius of 60m, and the axis of rotation is at a height of 75m above ground level. The problem can be solved by developing a formula, using either sine or cosine, to relate the height of a car above the ground to its rotation angle. The formula can then be used in a spreadsheet to create a table of results and graph:

	A	B
1	Angle	Height
2	0	12
3	10	9.916221868
4	20	7.89575828
5	30	6
6	40	4.286548684
7	50	2.807466683
8	60	1.607695155
9	70	0.723688551
10	80	0.182306964
11	90	0
12	100	0.182306964
13	110	0.723688551
14	120	1.607695155
15	130	2.807466683
16	140	4.286548684
17	150	6
18	160	7.89575828
19	170	9.916221868
20	180	12
21	190	14.08377813

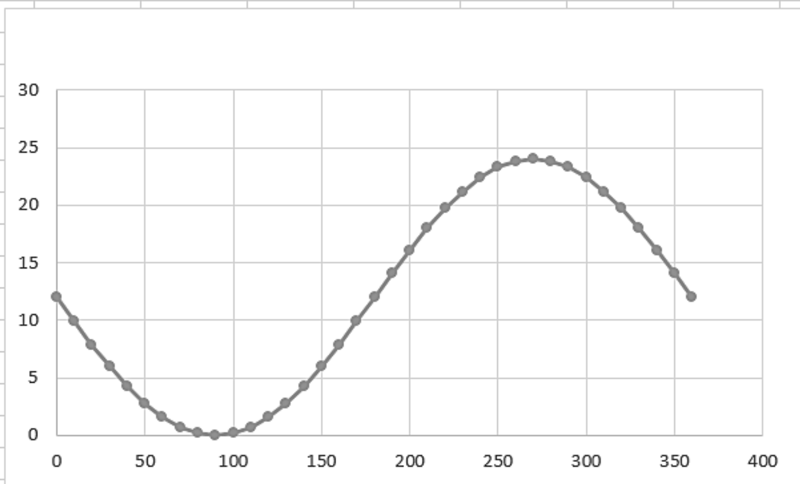


Figure 15: A possible graphical solution of the London Eye problem.

Rotation angles can be converted to minute times during the rotation, using the information that one rotation of the wheel takes approximately 30 minutes.

It is possible to estimate the height of the County Hall building from a photograph of the wheel. A line can then be constructed on the graph to represent the required height for photography. The time period when the car is above this level can then be found. The result should be given to a reasonable degree of accuracy, bearing in mind that much of the data used in the calculation was only approximate.

Special subject

Students who have studied a vocational topic are given the opportunity to investigate the topic further through a quantitative project. The features of special subject task are:

- The objective is to make a cross connection of knowledge: the teacher, together with students, solves the application problems
- The project includes a review and summary of the subject area.
- Students' ability to experiment mathematically is encouraged by use of modern technology such as a calculator or computer.
- The problems dealt with are more difficult and more open than is the case for extension problems.
- Students put forward and test a model hypothesis.
- Students complete the whole process of solving the problem independently and cooperatively. This represents a transition from teacher-centred to student-centred learning.
- Special subject problems still do not represent the total process of mathematical modelling

This approach was used successfully with construction students who had been studying heat losses from buildings, and gave a deeper understanding of the mathematics involved.

> After discussion of the insulating properties of different building components, students should develop their own spreadsheets to determine the heat losses from a house. This should allow investigation of the effects of double glazing of windows, cavity insulation of walls, and insulation of the roof space.

U-values measure the effectiveness of a material as an insulator in buildings. The units are Watts per square metre. A Watt is a unit of *power*, measured as joules of *energy* per second. U-values express the rate at which heat energy is transferred through each square metre of building component for each degree of temperature difference.

The lower the U-value, the better the material performs as a heat insulator. Some typical U-values for building materials are:

- a cavity wall has a U-value of 1.6 $W/m^2\ ^oC$
- a solid brick wall has a U-value of 2.0 $W/m^2\ ^oC$
- a double glazed window has a U-value of 2.8 $W/m^2\ ^oC$

Most heat is lost from homes through the windows and roof. Heat losses can be reduced by installing double glazing and loft insulation. The heat losses through outside cavity walls can be reduced by inserting cavity insulation. These techniques can produce savings on energy bills, but do themselves have an initial installation cost which may be quite high. It can be important to carry out calculations to determine the relative costs, before going ahead with insulation schemes.

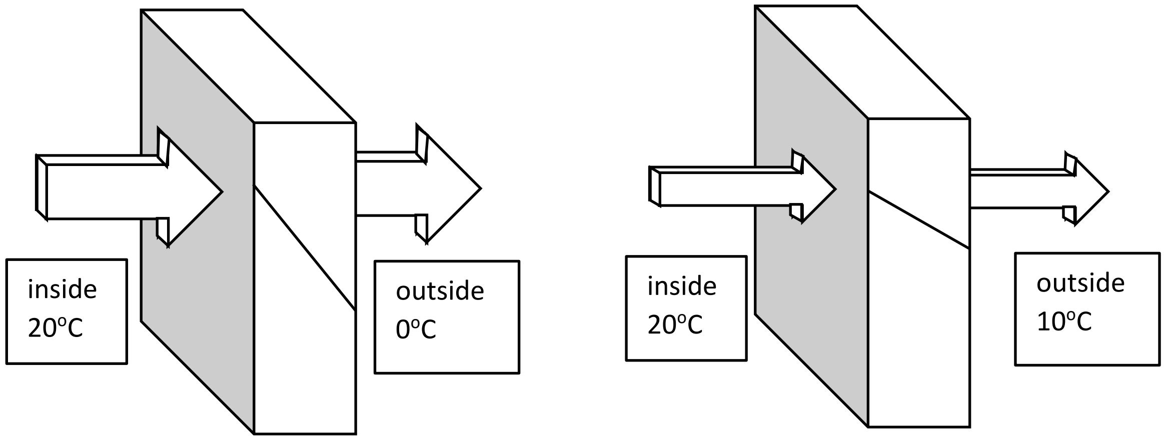


Figure 16: Temperature gradient through the wall of a building

Rate of heat transfer is proprtional to the temperature difference between the inside and outside surfaces of the wall. The heat transmission through a building wall **H** in **watts** is given by:

$$H = U\,A\,dt$$

where **U** is the U-value in **W/m²**
A is the wall area in **m²**
dt is the temperature difference across the wall surfaces in **°C**

A spreadsheet can be developed to carry out heat loss calculation. The stages of the calculation are:

1. Create input cells for room dimensions, allowing for door and wall openings. The height of the room needs to be specified. Cells where data can be added are shaded in colour.

	A	B	C	D	E	F
1	House heating calculations					
2						
3				length	height	width
4	Room 1	wall1	wall	5	3	
5	Dining room	Back wall	window	3	2	
6			door			
7		wall2	wall	4	3	
8		Party wall	window			
9			door	1	2	
10		wall3	wall	5	3	
11		Lounge wall	window			
12			door			
13		wall4	wall	4	3	
14		Kitchen wall	window			
15			door			
16		wall5	wall			
17			window			
18			door			
19		wall6	wall			
20			window			
21			door			
22		floor		5		4
23		ceiling		5		4

Areas of walls, floor, ceiling, windows and doors can then be calculated. The wall area is found in each case by subtracting the area of any window or door.

B	C	D	E	F	G
ting calculations					
		length	height	width	area
wall1	wall	5	3		9
Back wall	window	3	2		6
	door				0
wall2	wall	4	3		10
Party wall	window				0
	door	1	2		2
wall3	wall	5	3		15
Lounge wall	window				0
	door				0
wall4	wall	4	3		12
Kitchen wall	window				0
	door				0
wall5	wall				0
	window				0
	door				0
wall6	wall				0
	window				0
	door				0
floor		5		4	20
ceiling		5		4	20
room volume	45	5	3	3	

2. A lookup table of typical U-values is created.

Q	R	S
U-value code	U-value	
0	0	
1	0.18	external wall
2	0.39	internal wall
3	0.38	party wall
4	0.29	ceiling/floor
5	0.05	ceiling to roof
6	0.12	floor
7	0.51	window
8	0.42	door

3. We can now specify the required temperature for the room in °C, and the approximate temperature on the other side of each wall, floor and ceiling. The spreadsheet can then calculate the temperature difference.

B	C	D	E	F	G	H	I	J
ng calculations								
		length	height	width	area	inside temp	outside temp	temp diff
wall1	wall	5	3		9	20	10	10
Back wall	window	3	2		6	20	10	10
	door				0	20	10	10
wall2	wall	4	3		10	20	15	5
Party wall	window				0	20	15	5
	door	1	2		2	20	15	5
wall3	wall	5	3		15	20	15	5
Lounge wall	window				0	20	15	5
	door				0	20	15	5
wall4	wall	4	3		12	20	15	5
Kitchen wall	window				0	20	15	5
	door				0	20	15	5
wall5	wall				0	0		0
	window				0	0	0	0
	door				0	0	0	0
wall6	wall				0	0		0
	window				0	0	0	0
	door				0	0	0	0
floor		5		4	20	20	10	10
ceiling		5		4	20	20	10	10

4. The lookup table is used to select appropriate U-values. The user can specify the code number for each building component, as listed in the lookup table, and the spreadsheet will retrieve the corresponding U-value. Heat loss can then be calculated, as the product of the area, U-value and temperature difference.

G	H	I	J	K	L	M	N
area	inside temp	outside temp	temp diff	U-value code	U-value	Loss (watts)	
9	20	10	10	1	0.18	16.2	
6	20	10	10	7	0.51	30.6	
0	20	10	10		0	0.0	
10	20	15	5	3	0.38	19.0	
0	20	15	5		0	0.0	
2	20	15	5		0	0.0	
15	20	15	5	2	0.39	29.3	
0	20	15	5		0	0.0	
0	20	15	5		0	0.0	
12	20	15	5	2	0.39	23.4	
0	20	15	5		0	0.0	
0	20	15	5		0	0.0	

Q	R	S
U-value code	U-value	
0	0	
1	0.18	external wall
2	0.39	internal wall
3	0.38	party wall
4	0.29	ceiling/floor
5	0.05	ceiling to roof
6	0.12	floor
7	0.51	window
8	0.42	door

5. Heat loss through the walls, windows, doors, floor and ceiling of the room can then be added to calculate the rate at which energy is being lost in **watts** of power (**joules per second**). Assuming that this rate of energy loss is constant, it can be expressed as the amount of energy consumed each hour:

power (**W**) x time (**hours**) / 1000 = energy consumption (**kWh**)

A further, very important, factor in calculating the heat loss from a room is **ventilation**. Rooms become uncomfortable if fresh air is not circulating. It is recommended that the air in a room is changed twice each hour. If warmer air leaves the room and cooler air enters, heating will be needed to compensate for the heat loss.

We can calculate the volume of air entering the room each second by dividing the hourly volume by the number of seconds in an hour:

air volume per second (**m^3/sec**) = room volume (**m^3**) x air changes per hour / 3600

The volume of air can then be converted to a mass using the approximate air density figure of 1.225 kg/m^3.

We can now find the heat lost each second by exchange with cooler air. We multiply the mass of air entering the room by the temperature difference, then multiply by the specific heat capacity of air to convert to energy.

heat loss (**kJ**) = mass of air (**kg**) x temperature change (**°C**) x heat capacity of air (**kJ/kg °C**)

The specific heat capacity of air has a value of approximately 1.0

	5		4	20	20	10	10	6	0.12	24.0	
	5		4	20	20	10	10	4	0.29	58.0	
45	5	3	3		air changes per hour		2	air exhange temperature		15	
							room fabric heat loss/hour: kWh			0.2	
							air change: kg/hour			110.3	
							air change heat loss: kWh			0.2	
							total heat loss/hour			**0.4**	**kWh**

Rows of the spreadsheet can be copied to allow input of data and calculations for additional rooms of the house.

An estimate for the total heat loss from the building can be found. This would need to be replaced by a heating system. The price of electricity, gas or other fuel could then be used to estimate the heating cost.

The spreadsheet can be used to compare heating costs in summer and winter, and to experiment with different thermal insulation methods such as: cavity wall insulation, double grazing, or loft insulation.

Investigation Report

For this approach, students gather their own primary data through surveys, laboratory or fieldwork measurements, then process the data using appropriate mathematical methods. In this way, it is hoped to gain a clearer interpretation of the data and to obtain insights which were not initially obvious from qualitative observations. The features of an investigation report are:

- The task combines class teaching with activity outside of class
- By investigations, data collection and problem solving outside of class, students learn how to think about the world in terms of mathematics
- Where possible, students experience the use of computer technology to process and analyse their data
- Data collection activities provide an environment in which students can work together effectively, analysing problems and deciding on the sharing of tasks
- Practical data collection makes students go outside the college and puts them into an interesting, varied and dynamic learning environment
- The teacher determines the context and framework for the investigation, ensuring that the research carried out by students will lead to the solution of a worthwhile practical problem
- The teacher's specification of the problem places a restriction on students' thought processes, and inevitably limits the openness of the project.

As an example, geography students investigate coastal processes by measuring pebbles at different points along the Ro Wen shingle spit at Fairbourne, on the coast of Cardigan Bay.

Figure 17: The Ro Wen shingle spit, looking south in the direction of Friog

The Ro Wen shingle spit has been built up across the mouth of the Mawddach estuary by the process of longshore drift carrying pebbles northwards around the coast of Cardigan Bay. The source of most of the pebbles is from the erosion of soft boulder clay cliffs, seen in the distance in Figure 17.

Although a mixture of pebble sizes are present at each location along the spit, there is a reduction in mean size with distance from the cliffs at Friog. Research from other coastal locations has shown that two mechanisms may be affecting pebble size distribution:

Bartholoma, Ibbeken and Schleyer (1998), from a study of **Bianco Beach, Calabria**, southern Italy, found that attrition by waves during high energy storms caused pebbles to be broken down to smaller sizes.

Bird (1996), in a study of **Chesil Beach in Dorset**, found that pebbles of different sizes were transported along the beach at different rates by longshore drift, leading to the development of size gradation.

Students visited the Ro Wen shingle spit and made measurements of the mean dimension of pebbles on the upper storm beach at a series of locations within three areas: close to the Friog cliffs at the start of the spit, half way along the spit, and close to the end of the spit at the mouth of the Mawddach estuary at Barmouth Ferry.

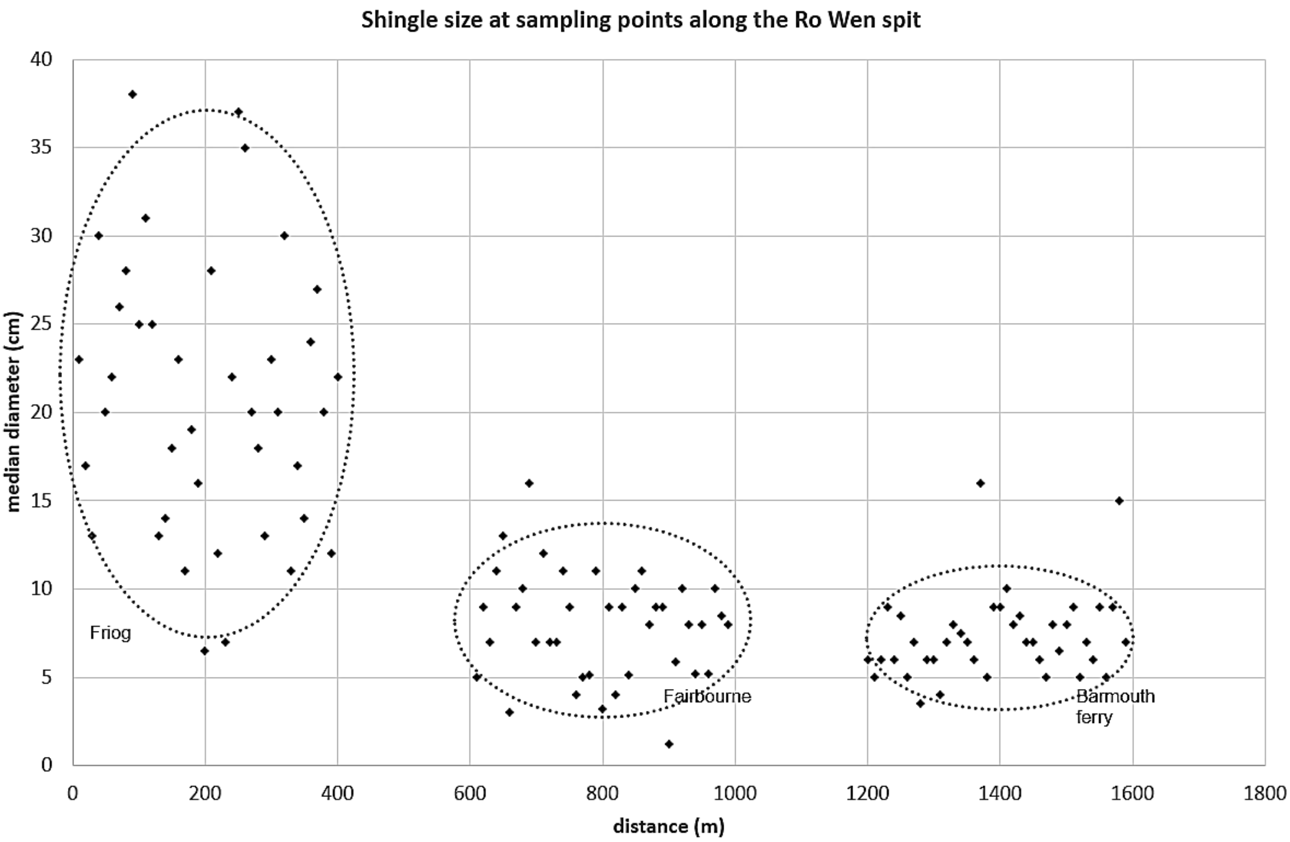


Figure 18: Pebble size sampling along the Ro Wen shingle spit

It is seen that a range of sizes occur at each location but the mean size appears to decrease along the length of the spit, quickly at first than more gradually. Combined with this effect is an increase in the sorting of the material.

A hypothesis was developed that the rate of size reduction along the length of the shingle spit would be related to the actual pebble size.

- smaller pebbles are progressively moved along the spit, with rate of movement proportional to size. Large pebbles are rarely in motion except during large storms, so remain at the start of the spit.
- large pebbles would be eroded more easily by wave action than small pebbles, so are reduced in size more rapidly,

The **exponential function** describes the mathematical situation in which the **change in a quantity** is **proportional to the quantity itself**. Experimenting with the fitting of trend lines in an Excel spreadsheet for different mathematical functions, it is found that an exponential function is indeed to best fit of curve for the experimental data.

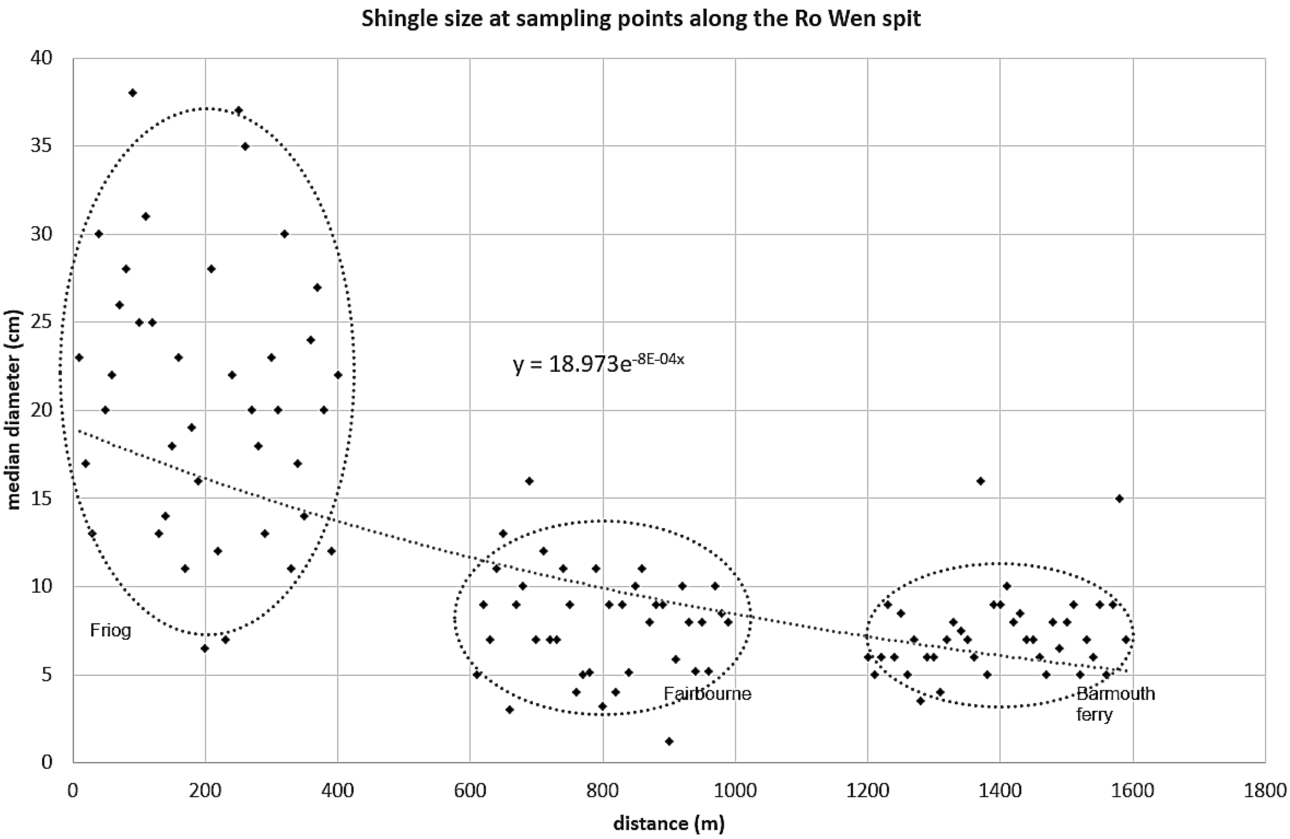


Figure 19: Exponential curve fitted to the distribution of pebble sizes along the Ro Wen shingle spit

The exponential function represents the differential equation:

$$\frac{d\ (size)}{d\ (distance)} = constant\ \times size$$

The solution of this differential equation is:

$$size = A\,e^{B \times distance}$$

The spreadsheet software has calculated the numerical values A = 18.973 and B = - 0.0008 as the values for the best fit curve.

Students were able to further explore the mathematical relationship by creating a theoretical model in which pebbles were reduced by a fixed fraction of their size for each distance unit along the shingle spit.

	A	B	C
1	reduction factor		0.992
2			
3	0	20	
4	10	19.84	
5	20	19.68128	
6	30	19.52383	
7	40	19.36764	
8	50	19.2127	
9	60	19.059	
10	70	18.90652	
11	80	18.75527	
12	90	18.60523	
13	100	18.45639	
14	110	18.30874	
15	120	18.16227	
16	130	18.01697	
17	140	17.87283	
18	150	17.72985	
19	160	17.58801	
20	170	17.44731	
21	180	17.30773	
22	190	17.16927	
23	200	17.03191	
24	210	16.89566	
25	220	16.76049	

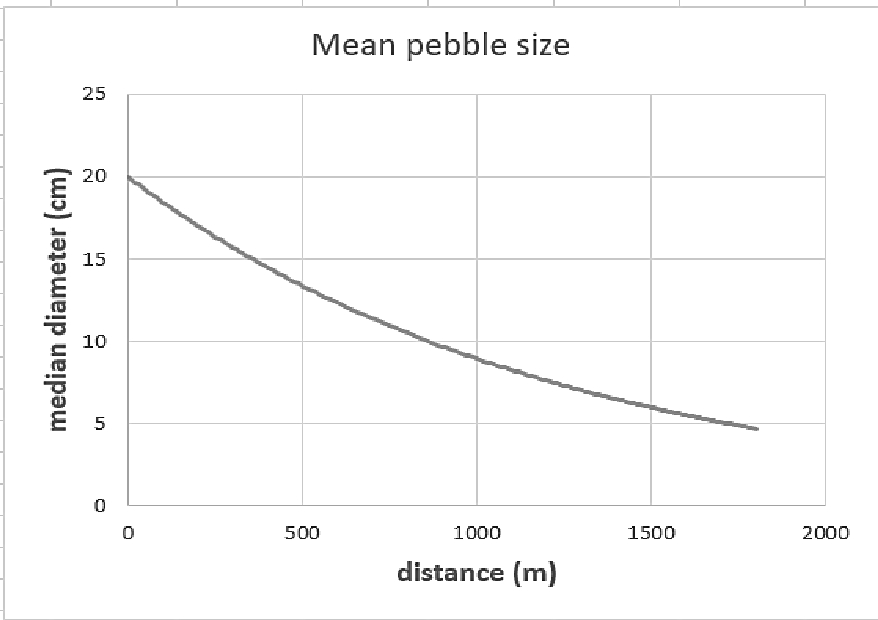


Figure 20: Exponential model for pebble size

Experimentation showed that a size reduction factor of 0.992 for each 10 metres of transport distance gives a very similar curve to the exponential function fitted to the experimental data.

Paper Discussion

The next approach described is to present students with an interesting and challenging vocational mathematics task, then provide resources from books, journal articles or the Internet which will allow the students to teach themselves the necessary quantitative techniques for solving the problem. This is deliberately intended to encourage students to develop as independent learners, and contrasts with the normal teaching approach in which the tutor provides the instruction. The features of a paper discussion are:

- The teacher chooses the papers carefully, students read them outside class and discuss them in class, perhaps led by one of the students
- Students learn about mathematical applications and modelling by studying examples produced by expert practitioners
- A major objective is to train students' independent learning abilities, and to make students appreciate the difference between mathematical papers and other articles
- Students have an opportunity to consider the background to the problem, the modelling methods, the solution tactics, and the solution methods
- The paper discussion method can break the traditional pattern in which students learn directly from the teacher. The independent learning method is more active, flexible and open.

An example project which might be chosen is the modelling of fish stocks. Students are provided with an article by Bourne (2012) entitled 'Collapsing fish stocks'. There have been some well-documented collapses in fish stocks, including:

- Atlantic cod, which went from a sustainable rate of 250,000 tonnes per year in the late 1950s, to an unsustainable 800,000 tonnes in 1968, and then collapsed to 1,700 tonnes by 1995.
- Norwegian herring, which collapsed in the late 1960s and the fishery was closed from 1977 to 1981.

The first model outlined by Bourne is for the establishment of a stable fish population, based on a differential equation

Differential equations involve an instantaneous time rate of change term, $\frac{dx}{dt}$ where x is the amount of fish in the sea (as a mass, or possibly as a gross number of fish), and t is the time (usually in months or years).

Built in to each fish stock equation is a positive (growth) term (dependent on food supply, breeding rates, etc), and a negative (inhibition) term (due to limits on food availability, etc).

Here's a highly simplified example of a model for the amount of fish expected in a particular area bounded by some fixed geographic boundary (for example, a bay):

$$\frac{dx}{dt} = x\left(1 - \frac{x}{12}\right)$$

The growth term is the x outside the brackets on the right hand side, and the negative inhibition term is contained in the brackets. The number "12" is important, since this indicates a stable value for the amount of fish, all else being equal.

We need a **starting value**. In this example, for illustration, we start with 500 tonnes of fish in the sea, and we write it like this (our unit for mass of fish throughout the equations and graphs in this discussion is "thousand tonnes"):

$$x(0) = 0.5$$

A precise solution to this differential equation can be obtained analytically, and would provide an interesting exercise for A-level mathematics students. However, our purpose here is to focus on interpreting the results of the mathematical model, so a fairly accurate solution by a numerical method will be adequate and is easier to understand.

Our approach will use Euler's method to produce a graph for the differential equation:

$$\frac{dx}{dt} = x\left(1 - \frac{x}{12}\right)$$

We will assume that **x** represents thousands of tonnes of fish, and **t** represents years after the commencement of the model. Euler's method allows the quantity at each successive time interval to be equal to the previous quantity plus the change which occurs:

$$x_{t+1} = x_t + t\frac{dx}{dt}$$

Starting at year t = 0, this allows us to calculate the amount of fish in each subsequent year by means of a recurrence relation:

$$x_{t+1} = x_t + x_t\left(1 - \frac{x_t}{12}\right)$$

The weight of fish in year 0 is given as 0.5 thousand tonnes.

Students create a graph of change in fish population over time. It is found that the model becomes stable after about 8 years:

	A	B	C
1	time	quantity	dx/dt
2	0	0.5	0.479167
3	1	0.979167	0.899269
4	2	1.878436	1.584393
5	3	3.462829	2.463563
6	4	5.926392	2.999548
7	5	8.925941	2.286573
8	6	11.21251	0.735809
9	7	11.94832	0.051455
10	8	11.99978	0.000223
11	9	12	4.13E-09
12	10	12	0
13	11	12	0
14	12	12	0
15	13	12	0
16	14	12	0
17	15	12	0
18	16	12	0
19	17	12	0
20	18	12	0

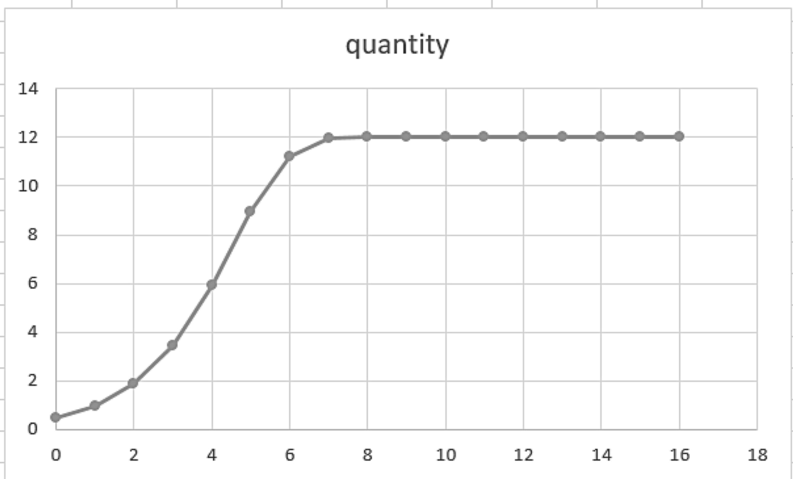


Figure 21: Model for a stable fish stock

In the stable case, the rate of change is zero, so

$$x\left(1-\frac{x}{12}\right)=0$$

This is true if either x = 0, which is the trivial case where there are no fish, or when x = 12.

Continuing to work through the paper by Bourne (2012), students can now examine a situation in which the initial fish population is greater than the available environmental resources can support:

> **Overpopulated case**
>
> Let's now look at the case where, for some reason (it could be a temporary increase in food supply), the amount of fish went beyond 12,000 tonnes. If the food supply then returns to normal, the bay will not be able to sustain so many fish.
>
> To illustrate, we let the initial condition be 14,000 tonnes of fish.
>
> $x(0) = 14$

Again using Euler's method but changing the initial quantity of fish in year 0, students obtain a graph which falls from 14,000 tonnes to the stable level of 12,000 tonnes.

	A	B	C
1	time	quantity	dx/dt
2	0	14	-0.23333
3	0.1	13.76667	-0.20268
4	0.2	13.56399	-0.17678
5	0.3	13.38721	-0.15476
6	0.4	13.23245	-0.1359
7	0.5	13.09655	-0.11967
8	0.6	12.97687	-0.10564
9	0.7	12.87123	-0.09345
10	0.8	12.77778	-0.08282
11	0.9	12.69496	-0.07352
12	1	12.62144	-0.06536
13	1.1	12.55608	-0.05818
14	1.2	12.4979	-0.05186
15	1.3	12.44604	-0.04626
16	1.4	12.39978	-0.04131
17	1.5	12.35847	-0.03692
18	1.6	12.32155	-0.03302
19	1.7	12.28853	-0.02955
20	1.8	12.25899	-0.02646
21	1.9	12.23253	-0.0237
22	2	12.20883	-0.02125
23	2.1	12.18758	-0.01905
24	2.2	12.16853	-0.01709
25	2.3	12.15144	-0.01534
26	2.4	12.1361	-0.01376
27	2.5	12.12234	-0.01236
28	2.6	12.10998	-0.0111
29	2.7	12.09888	-0.00997
30	2.8	12.08891	-0.00896

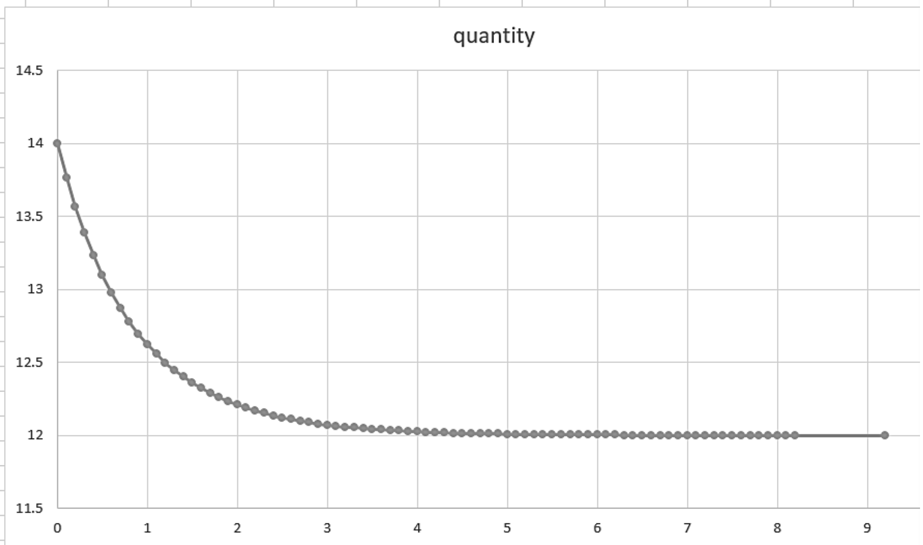


Figure 22: Model of fish stocks with initial overpopulation

Returning to Bourne's paper, students now examine the effects of fishing. A scenario is presented in which fish stocks will be exploited whilst allowing fishing to continue in a sustainable manner.

Sustainable fishing case

Next, let's assume a fishing trawler moves into the bay. They take out 2,000 tonnes of fish per year. (It's a small trawler. Typically trawlers can catch 4,000 to 5,000 tonnes per year).

Our differential equation becomes:

$$\frac{dx}{dt} = x\left(1 - \frac{x}{12}\right) - 2$$

We choose our starting value as the stable value of 12,000 tonnes.

$$x(0) = 12$$

Euler's method is again used, but the recurrence relation is changed to allow for the annual fish catch:

$$x_{t+1} = x_t + x_t\left(1 - \frac{x_t}{12}\right) - 2$$

	A	B	C
1	time	quantity	dx/dt
2	0	12	-2
3	1	10	-0.33333
4	2	9.666667	-0.12037
5	3	9.546296	-0.04802
6	4	9.498278	-0.01983
7	5	9.478449	-0.0083
8	6	9.470148	-0.00349
9	7	9.466654	-0.00147
10	8	9.46518	-0.00062
11	9	9.464557	-0.00026
12	10	9.464294	-0.00011
13	11	9.464183	-4.7E-05
14	12	9.464136	-2E-05
15	13	9.464116	-8.4E-06
16	14	9.464108	-3.5E-06
17	15	9.464104	-1.5E-06
18	16	9.464103	-6.3E-07
19	17	9.464102	-2.7E-07
20	18	9.464102	-1.1E-07
21	19	9.464102	-4.8E-08
22	20	9.464102	-2E-08

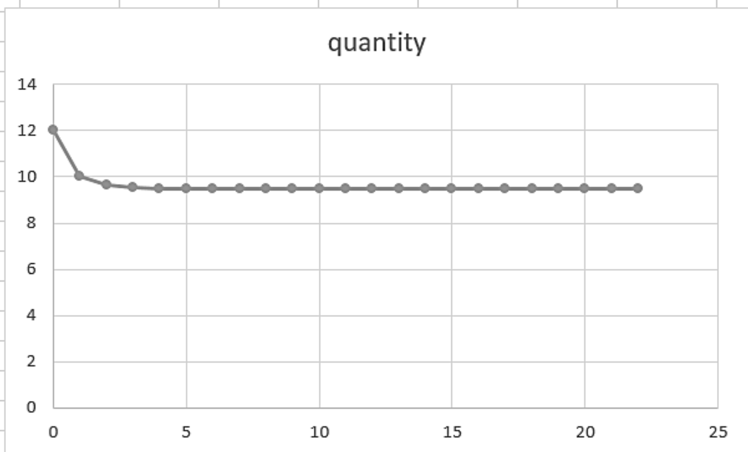


Figure 23: Model of sustainable fishing

It is found that a stable population is established at around 9,500 tonnes, which is only slightly lower than the population of 12,000 tonnes which occurs in the absence of fishing.

A final example by Bourne examines an unsustainable case, where overfishing can lead to the collapse of the fish stock.

Unsustainable fishing case 2

This time, let's start with a very robust amount of fish (20,000 tonnes) and use a fishing rate of 6,000 tonnes per year.

This is the differential equation for this situation, and the initial value.

$$\frac{dx}{dt} = x\left(1 - \frac{x}{12}\right) - 6$$

$$x(0) = 20$$

Students again modify the recurrence relation, then create the spreadsheet graph. It is found that changes to fish stocks occur very rapidly, so a shorter time interval of 0.1 years is used for the model. It will be necessary to introduce a factor of 0.1 in the recurrence relation to allow for this:

$$x_{t+1} = x_t + 0.1\left[x_t\left(1 - \frac{x_t}{12}\right) - 6\right]$$

A total collapse of fish stocks is found to occur after about 4 years.

	A	B	C
1	time	quantity	dx/dt
2	0	20	-1.93333
3	0.1	18.06667	-1.51337
4	0.2	16.5533	-1.2281
5	0.3	15.3252	-1.02466
6	0.4	14.30054	-0.87416
7	0.5	13.42638	-0.75959
8	0.6	12.66679	-0.67038
9	0.7	11.9964	-0.59964
10	0.8	11.39676	-0.54271
11	0.9	10.85405	-0.49635
12	1	10.3577	-0.45825
13	1.1	9.899458	-0.42671
14	1.2	9.472743	-0.4005
15	1.3	9.072243	-0.37866
16	1.4	8.693588	-0.36046
17	1.5	8.333126	-0.34536
18	1.6	7.987764	-0.33293
19	1.7	7.654837	-0.32282
20	1.8	7.332016	-0.31479
21	1.9	7.017231	-0.30862
22	2	6.708608	-0.30418
23	2.1	6.404423	-0.30136
24	2.2	6.10306	-0.30009
25	2.3	5.802972	-0.30032
26	2.4	5.502648	-0.30206
27	2.5	5.200587	-0.30533
28	2.6	4.895262	-0.31017
29	2.7	4.585091	-0.31668
30	2.8	4.268408	-0.32499

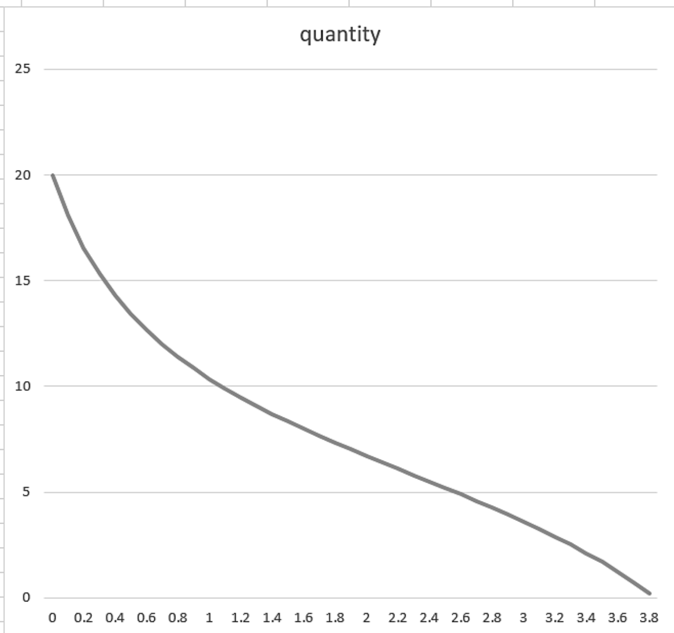


Figure 24: Model of unsustainable fishing

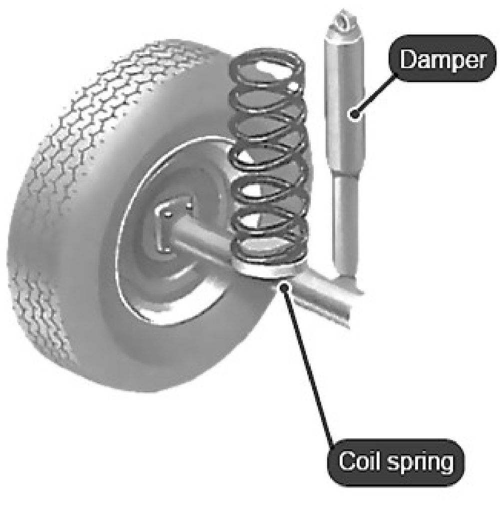
Mini Scientific Research

The final example given by Tang, Sui, & Wang (2003) is mini scientific research. This represents the maximum level of student involvement in the planning, investigation and analysis of data for a substantial numeracy project related to their vocational area. Features are:

- The teacher shows students examples of previous projects, and initiates discussion of interesting topics which might be investigated.
- Students make initial plans which outline the research question to be addressed, the types and amount of data to be collected, the mathematical approach to analysing the data, and the way in which the results will be presented.
- In consultation with the teacher, the students begin their research. Regular progress meetings are held, at which preliminary data and findings can be discussed, and plans made for the next stage of the project.
- The final results of the project will be presented in a formal meeting, and written up as a technical report.

An example project carried out by engineering students has been the investigation of the motion of a car when passing over a speed hump, in response to the springs and shock absorbers of the car suspension system. These components have different responses: forces are related to **change in length** for a **spring**, but to the **velocity of movement** for a **shock absorber**.

For simplicity, each car wheel is considered to have an independent suspension system. This will consist of a coil spring and a shock absorber.



www.howacarworks.com/illustrations/coil-spring

Figure 25: Components of a car suspension system

Using a system of one spring and shock absorber:

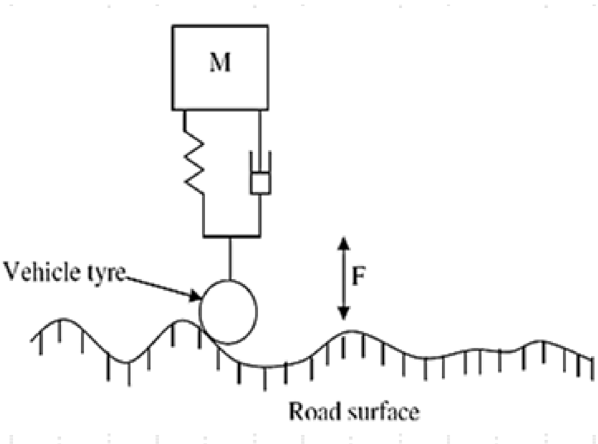


Figure 26: Initial simplified model for the car suspension

An algorithm for each time step can be constructed:

- Find the current vertical displacement of the car body from its rest position relative to the road surface, $\mathbf{x_n}$
- Use the displacement to calculate the spring force

$$\mathbf{s_n = k_s \,.\, x_n}$$

- Get the current vertical velocity of the car body, $\mathbf{v_n}$
- Use the vertical velocity to calculate the damping force

$$\mathbf{d_n = k_d \,.\, v_n}$$

- Combine the spring and damper forces to determine the resultant vertical force on the car body

$$\mathbf{f_n = s_n + d_n}$$

- Use the force to determine the vertical acceleration of the car body, given the mass of the car

$$\mathbf{a_n = f_n / m}$$

- Apply the acceleration for a time interval to determine the new vertical velocity

$$\mathbf{v_{n+1} = v_n + a_n \,.\, t}$$

- Apply the vertical velocity, change in length of the shock absorber, and change in road elevation, to determine the new displacement of the car body

$$\mathbf{x_{n+1} = x_n + v_{n+1} \,.\, t + d_n + r_n}$$

A spreadsheet can be constructed to implement the algorithm for a series of time steps:

	A	B	C	D	E	F	G	
1								
2		mass	10			mass*acceleration + constant*displacement = force		
3		force	10					
4		constant	2					
5		damp	0.9					
6								
7		time	displacement	acceleration	force	velocity	damping	forcing
8		0	0	=E8/mass	=-C8*constant	0	=F8*damp	0.1
9		=B8+1	=C8+F9-G8+H8	=E9/mass	=-C9*constant	=F8+D8	=F9*damp	0.2
10		=B9+1	=C9+F10-G9+H9	=E10/mass	=-C10*constant	=F9+D9	=F10*damp	0.3
11		=B10+1	=C10+F11-G10+H10	=E11/mass	=-C11*constant	=F10+D10	=F11*damp	0.4
12		=B11+1	=C11+F12-G11+H11	=E12/mass	=-C12*constant	=F11+D11	=F12*damp	0.3
13		=B12+1	=C12+F13-G12+H12	=E13/mass	=-C13*constant	=F12+D12	=F13*damp	0.2
14		=B13+1	=C13+F14-G13+H13	=E14/mass	=-C14*constant	=F13+D13	=F14*damp	0.1
15		=B14+1	=C14+F15-G14+H14	=E15/mass	=-C15*constant	=F14+D14	=F15*damp	0
16		=B15+1	=C15+F16-G15+H15	=E16/mass	=-C16*constant	=F15+D15	=F16*damp	
17		=B16+1	=C16+F17-G16+H16	=E17/mass	=-C17*constant	=F16+D16	=F17*damp	
18		=B17+1	=C17+F18-G17+H17	=E18/mass	=-C18*constant	=F17+D17	=F18*damp	

Initially a set of arbitrary values can be specified for the constants, in order to test that the model provides the correct qualitative responses:

	A	B	C	D	E	F	G	H	I
1									
2		mass	10			mass*acceleration + constant*displacement = force			
3		force	10						
4		constant	2						
5		damp	0.9						
6									
7		time	displacement	acceleration	force	velocity	damping	forcing	
8		0	0	0	0	0	0	0.1	
9		1	0.1	-0.02	-0.2	0	0	0.2	
10		2	0.28	-0.056	-0.56	-0.02	-0.018	0.3	
11		3	0.522	-0.1044	-1.044	-0.076	-0.0684	0.4	
12		4	0.81	-0.162	-1.62	-0.1804	-0.16236	0.3	
13		5	0.92996	-0.185992	-1.85992	-0.3424	-0.30816	0.2	
14		6	0.909728	-0.1819456	-1.819456	-0.528392	-0.4755528	0.1	
15		7	0.7749432	-0.15498864	-1.5498864	-0.7103376	-0.63930384	0	
16		8	0.5489208	-0.10978416	-1.0978416	-0.86532624	-0.778793616		
17		9	0.352604016	-0.070520803	-0.705208032	-0.9751104	-0.87759936		

The displacement–time graph does produce a realistic result for damped simple harmonic motion in response to the car travelling over a speed hump:

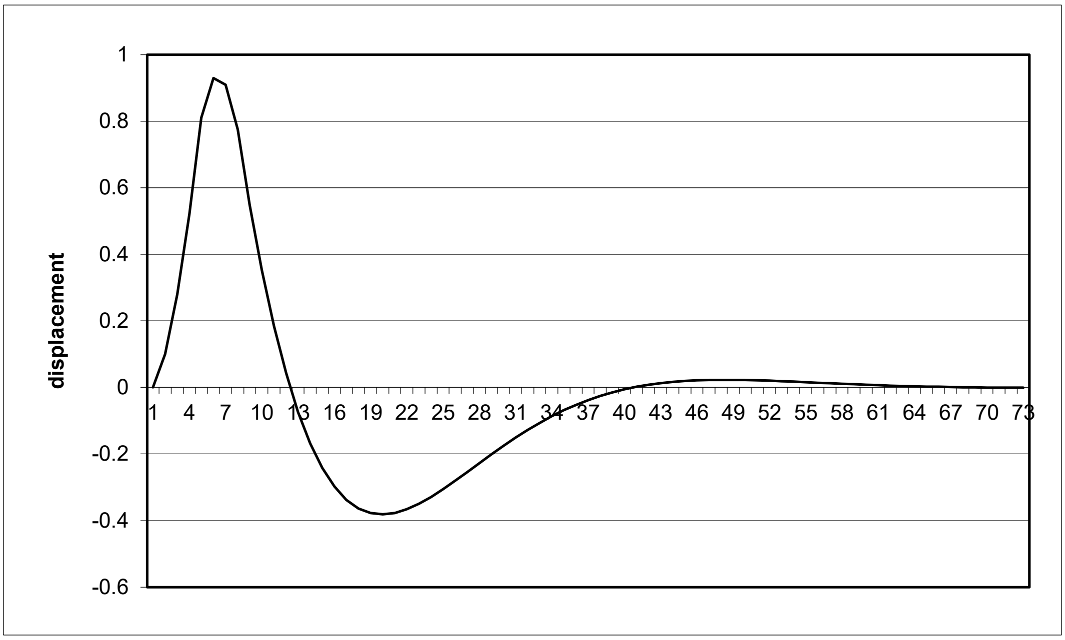


Figure 27: Displacement-time graph for the initial simplified model of the car suspension

Engineering students can continue to develop the model:

- Realistic values can be specified for the car mass, spring and damper constants
- The model can be extended to allow for the spring effect of the tyre
- Effects of different profiles of speed hump, and different car speeds, can be investigated.

Summary

In this chapter we have considered various ways in which numeracy activities can be integrated into college courses at level 3.

Where students are required to study Essential Skills, it can be effective to combine Application of Number, Communication, and Information and Communications Technology together in one or more integrated projects. This will allow students to select substantial and relevant tasks which interest them. Letting students have the autonomy to design their own projects, including the practical collection of data, can be motivating and encourages the development of problem solving skills.

We have found that it is best to initially design an integrated project around the Use of Number component. There will then be ample opportunities for demonstrating communication skills during the design process, and in presenting the final results. Computer skills can be developed during the data processing and presentation stages of the project.

Students can benefit, both in terms of motivation and preparation for employment, if interesting and realistic practical numeracy activities are integrated into their main vocational courses. We have outlined the framework proposed by Tang, Sui & Wang (2003) for the integration of numeracy at a series levels, representing a progression from applications set by the teacher, through increasing student involvement in the solution of real world problems, to totally independent project work:

> **Extension.** After studying a mathematical topic, students are presented with an ill-defined real world problem and must seek out additional data for its solution.
>
> **Special Subject**. Students who have studied a vocational topic are given the opportunity to investigate the topic further through a quantitative project specified by the teacher.
>
> **Investigation Report**. Students gather their own primary data through surveys, laboratory or fieldwork measurements, then process the data using appropriate mathematical methods.
>
> **Paper Discussion**. Students are presented with an interesting and challenging vocational mathematics task, then provided with resources from books, journal articles or the Internet. The students are encouraged to teach themselves the necessary quantitative techniques for solving the problem.
>
> **Mini Scientific Research**. This represents the maximum level of student involvement in the planning, investigation and analysis of data for a substantial numeracy project related to their vocational area.